B.Tech. (ME) 2nd Semester G-Scheme
Examination, May-2019

MATH-II

Paper-BSC-MATH-102-G

(Multivariables Calculus, Differential Equations and
Complex Analysis)

Time allowed : 3 hours]   [Maximum marks : 75

Note : *Question No. 1 is compulsory. Attempt five questions
in total by selecting **one** question from each unit.
All questions carry equal marks.*

1.   (a)   Evaluate $\int\limits_{1}^{2}\left[\int\limits_{3}^{4}(xy + e^{y})dx\right]dy$.

     (b)   Solve $(2x\cos y + 3x^2y)\,dx +$
           $(x^3 - x^2\sin y - y)\,dy = 0$

     (c)   Define Mobius transformation, and when it is
           called univalent.

     (d)   prove that sinh z is analytic function.

     (e)   If n is an integer s.t. $n \neq -1$ and C is circle
           $|z - a| = r$. Then prove $\oint\limits_{C}(z - a)^n dz = 0$

     (f)   Evaluate $\oint\limits_{C}(x^2 - y^2 + 2ixy)$, where C is the
           contour $|z| = 1$.   6×2.5=15

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Unit-I

2. (a) Evaluate $\iint \frac{(x-y)^2}{x^2+y^2}\, dx\, dy$, over the circle $x^2 + y^2 \leq 1$. 7.5

(b) By changing the order evaluate the integral

$$\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\, dx\, dy.$$ 7.5

3. Verify Stoke's Theorem for $\vec{f} = x^2\hat{i} + xy\hat{j}$, integrated around the square in the plane $z=0$, whose sides are along the lines $x = 0$, $x = a$, $y = 0$ and $y = a$. 15

Unit-II

4. (a) Solve the equation $\frac{d^2y}{dx^2} + y\, \text{cosec}\, x$ by using method of variation of parameters. 7.5

(b) Solve Cauchy-Euler equation:

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x).$$ 7.5

5. (a) Express the polynomial x^3+2x^2-x-3 in terms of Legendre's polynomials. 7

(b) Find the power series solution about $x=0$, of $(1-x^2)\, y'' - 2xy' + 2y = 0$. 8

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Unit-III

6. State and prove necessary and sufficient conditions for f(z) to be analytic. 15

7. (a) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Find the conjugate function v and express u + iv as an analytic function of z. 8

   (b) Determine the analytic function whose real part is $(e^x x \cos y - e^x y \sin y)$. 7

Unit-IV

8. (a) Expand $\dfrac{e^{2z}}{(z-1)^3}$ in Laurent's series about its singularity. 7.5

   (b) Evaluate the residues of $\dfrac{z^2}{(z-1)(z-2)(z-3)}$ at z=1,2,3 and ∞, also determine their sum. 7.5

9. (a) Verify Cauchy's integral theorem by integrating e^{iz} along the boundary of the triangle with vertices at the points 1+i, −1+i and −1−i. 8

   (b) Use Cauchy's integral formula to evaluate

   $\oint_C \dfrac{e^{2z}}{(z+1)^4(z+5)} \, dz$, where C is the circle $|z| = 2$. 7

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